# LEARNING ACTIVE INFERENCE MODELS OF PERCEPTION AND CONTROL: APPLICATION TO CAR FOLLOWING TASK 

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## A Preview of Results



Figure 1: Visualizations of active inference model.

## Outline

(1) Modeling Perception and Control
(2) Learning a Model of Perception and Control
(3) Active Inference
4. Application to Car Following Task
(5) Conclusions

## Outline

(1) Modeling Perception and Control

## A POMDP Model

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- A generative model of observations $\mathbb{T}\left(o_{t} \mid s_{t}\right)$.
- A belief distribution about the hidden state $b_{t}(s)=\mathbb{P}\left(s_{t}=s \mid h_{t}\right)$
- A representation of state dynamics, i.e. a transition to a new state $s_{t+1}$ takes place with probability $\mathbb{P}\left(s_{t+1} \mid s_{t}, a_{t}\right)$


## A POMDP Model

- After $t>0$ time periods, the observable history of observations and actions is denoted by

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h_{t}:=\left\{o_{t}, \ldots, o_{0}, a_{t-1}, \ldots, a_{0}\right\} \in H_{t}
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- Denoting control policies (possibly random) by $\pi\left(\cdot \mid h_{t}\right)$, the POMDP model is the solution to:

$$
\max _{\pi} \mathbb{E}\left[\sum_{t \geq 0} \gamma^{t}\left[r\left(s_{t}, a_{t}\right)-c\left(\pi\left(\cdot \mid h_{t}\right)\right)\right]\right]
$$

where $r\left(s_{t}, a_{t}\right)$ is the reward and $c\left(\pi\left(\cdot \mid h_{t}\right)\right)$ information processing cost.

## A Bayesian Agent

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- When implementing action $a_{t}$ under beliefs $b_{t}$, the agent expects:
- a reward

$$
r\left(b_{t}, a_{t}\right):=\sum_{s} r\left(s, a_{t}\right) b_{t}(s)
$$

- observation $o_{t+1}$ with probability:

$$
\sigma\left(o_{t+1} \mid b_{t}, a_{t}\right):=\sum_{s_{t+1}} \sum_{s_{t}} \mathbb{T}\left(o_{t+1} \mid s_{t+1}\right) \mathbb{P}\left(s_{t+1} \mid s_{t}, a_{t}\right) b_{t}\left(s_{t}\right)
$$

## A Bayesian Agent

- With Markovian dynamics and additive reward the model of optimal behavior has recursive structure:

$$
\begin{aligned}
& V^{*}(b)=\max _{\pi(\cdot \mid b)}\left\{\sum_{s} \sum_{a} r(s, a) \pi(a \mid b) b(s)-c(\pi(\cdot \mid b))\right. \\
&\left.+\gamma \sum_{a} \sum_{o^{\prime}} \sigma\left(o^{\prime} \mid b, a\right) \pi(a \mid b) V^{*}\left(b^{\prime}\right)\right\}
\end{aligned}
$$

where $b^{\prime}$ is the resulting belief when observation $o^{\prime}$ is recorded after implementing action $a$.

## A Bayesian Agent

- With the information processing cost as Kullback-Leibler divergence between the control policy and a default policy $\pi^{0}$, i.e.

$$
c(\pi(\cdot \mid b))=\mathcal{D}_{K L}\left(\pi(\cdot \mid b) \| \pi^{0}(\cdot \mid b)\right)
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- The model is of the form:

$$
\begin{equation*}
\pi^{*}(a \mid b)=\frac{\pi^{0}(a \mid b) \exp \left(Q^{*}(b, a)\right)}{\sum_{a^{\prime} \in A} \pi^{0}\left(a^{\prime} \mid b\right) \exp \left(Q^{*}\left(b, a^{\prime}\right)\right)} \tag{1}
\end{equation*}
$$

where

$$
\begin{equation*}
Q^{*}(b, a):=r(b, a)+\gamma \sum_{o^{\prime}} \sigma\left(o^{\prime} \mid b, a\right) V^{*}\left(b^{\prime}\right) \tag{2}
\end{equation*}
$$

## Outline

(2) Learning a Model of Perception and Control

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- Perception The agent's internal representation: $\mathbb{P}_{\theta_{1}}\left(s^{\prime} \mid s, a\right)$ and $\mathbb{T}_{\theta_{1}}\left(o^{\prime} \mid s^{\prime}\right)$ parametrized by $\theta_{1} \in \mathbb{R}_{1}^{p}$.


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- Preferences A reward function $r_{\theta_{2}}(b, a)$ which is parametrized by $\theta_{2}$


## Learning a Model of Perception and Action

The log-likelihood of dataset $\mathcal{D}$ can be written as:

$$
\begin{aligned}
\log \mathbb{P}(\mathcal{D} \mid \theta) & =\log \prod_{\tau \in \mathcal{D}} \mathbb{P}(\tau \mid \theta) \\
& =\mathbb{E}_{\tau \sim \mathcal{D}}\left[\sum_{t=0}^{T} \log \left(\pi_{\theta}^{*}\left(a_{t} \mid b_{\theta_{1}, t}\right) \mathbb{P}\left(o_{t+1} \mid h_{t} \cup\left\{a_{t}\right\}\right)\right)\right]|\mathcal{D}| \\
& =\mathbb{E}_{\tau \sim \mathcal{D}}\left[\sum_{t=0}^{T} \log \pi_{\theta}^{*}\left(a_{t} \mid b_{\theta_{1}, t}\right)\right]|\mathcal{D}|+\text { constant }
\end{aligned}
$$

## Learning a Model of Perception and Action

Assumption 1: $P(\theta)=P\left(\theta_{1}\right) P\left(\theta_{2}\right)$, where:

$$
P\left(\theta_{1}\right) \propto \exp \left(\lambda \mathbb{E}_{\tau \sim \mathcal{D}}\left[\prod_{t=0}^{T} \sigma_{\theta_{1}}\left(o_{t+1} \mid b_{\theta_{1}, t}, a_{t}\right)\right]|\mathcal{D}|\right)
$$

for some $\lambda>0$.

## Learning a Model of Perception and Action

Assuming a uniform prior $P\left(\theta_{2}\right)$ on a compact subset $\Theta_{2} \subset \mathbb{R}_{2}^{p}$, the $\log$ of the posterior distribution can be written as:

$$
\begin{aligned}
& \log P(\theta \mid \mathcal{D})= \log P(\mathcal{D} \mid \theta)+\log P\left(\theta_{1}\right)+\text { constant } \\
&= \\
&=\mathbb{E}_{\mathcal{D}}\left[\log \sum_{t=0}^{T} \pi_{\theta}^{*}\left(a_{t} \mid b_{\theta_{1}, t}\right)+\lambda \sum_{t=0}^{T} \log \sigma_{\theta_{1}}\left(o_{t+1} \mid b_{\theta_{1}, t}, a_{t}\right)\right]|\mathcal{D}| \\
& \quad+\text { constant }
\end{aligned}
$$

## Learning a Model of Perception and Action

The estimation problem as the following bi-level optimization problem:

$$
\begin{aligned}
\max _{\left(\theta_{1}, \theta_{2}\right)} & \mathbb{E}_{\mathcal{D}}\left[\log \sum_{t=0}^{T} \pi_{\theta}^{*}\left(a_{t} \mid b_{\theta_{1}, t}\right)+\lambda \sum_{t=0}^{T} \log \sigma_{\theta_{1}}\left(o_{t+1} \mid b_{\theta_{1}, t}, a_{t}\right)\right] \\
\text { s.t. } & \pi_{\theta}^{*}=\arg \max _{\pi \in \Pi^{H}} \mathbb{E}\left[\sum_{h \leq H}\left[r_{\theta}\left(b_{h}, a_{h}\right)-\log \pi\left(\cdot \mid b_{h}\right)\right]\right]
\end{aligned}
$$

## Outline

(3) Active Inference

## Active Inference and Free Energy

## ACTIVE INFERENCE <br> The Free Energy Principle in Mind, Brain, and Behavior



THOMAS PARR GIOVANNI PEZZULO

KARL J. FRISTON
"Probably the most lucid and comprehensive treatment of the concept of active inference to date

> Active inference is a novel framework for cognition and behavior according to which the agent jointly perceives and acts upon the world so as to maximize the match between perceived vs preferred states of the world.

## Active Inference and Free Energy



A principle of free energy minimization:

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- (backward) free energy is minimized when the agent's belief distribution $b_{t}$ corresponds to the Bayes updated belief distribution on the state $s_{t}$.


## Active Inference and Free Energy



A principle of free energy minimization:

- (backward) free energy is minimized when the agent's belief distribution $b_{t}$ corresponds to the Bayes updated belief distribution on the state $s_{t}$.
- (forward) surprise is measured with respect to a preferred distribution $\tilde{P}\left(s_{t+1}\right)$ over states of the environment.


## Active Inference and Free Energy

The immediate "surprise" associated with action $a_{t}$ when current beliefs are $b_{t}$ is quantified by the expected free energy defined as:

$$
\operatorname{EFE}\left(b_{t}, a_{t}\right)=\mathbb{E}\left[D_{K L}\left(b_{t+1} \| \tilde{P}\right)\right]+\mathbb{E}\left[\mathcal{H}\left(\mathbb{T}\left(\cdot \mid s_{t+1}\right)\right)\right]
$$

where

$$
b_{t+1}(s)=\mathbb{P}\left(s_{t+1}=s \mid h_{t} \cup\left\{a_{t}, o_{t+1}\right\}\right)
$$

and $\mathcal{H}\left(\mathbb{T}\left(\cdot \mid s_{t+1}\right)\right)$ is the entropy of the resulting generative model of observations, i.e.:

$$
\mathcal{H}\left(\mathbb{T}\left(\cdot \mid s_{t+1}\right)\right):=-\sum_{o^{\prime}} \mathbb{T}\left(o^{\prime} \mid s_{t+1}\right) \log \left(\mathbb{T}\left(o^{\prime} \mid s_{t+1}\right)\right)
$$

## Outline

(4) Application to Car Following Task

## Application to Car Following Task, Ran et al. (2023)

- We use the active inference specification (reward equal to negative free energy).
- We use the INTERACTION dataset: a set of time-indexed trajectories of the positions, velocities, and headings of each vehicle in the scene in the map's coordinate system at a sampling frequency of 10 Hz .


## Application to Car Following Task



Figure 2: Top down view of the roadway in Dataset

## Application to Car Following Task



Figure 3: Offline evaluation MAE-IQM. Each point corresponds to a random seed used to initialize model training and its color corresponds to the testing condition of either same-lane or new-lane.

## Application to Car Following Task



Figure 4: Online evaluation ADE-IQM. Each point corresponds to a random seed used to initialize model training and its color corresponds to the testing condition of either same-lane or new-lane.

## Application to Car Following Task



Figure 5: Visualizations of a same-lane offline evaluation trajectory

## Application to Car Following Task



Figure 6: Visualizations of a same-lane online evaluation trajectory where the AIDA generated a rear-end collision with the lead vehicle.

## Outline

(5) Conclusions

## Conclusions

- We proposed a novel model of driver behavior using active inference (AIDA).
- Using car following data, we showed that the AIDA significantly outperformed the rule-based IDM on all metrics and performed comparably with the data-driven neural network benchmarks.
- We showed that the structure of the AIDA provides superior interpretability of its input-output mechanics than the neural network models.
- Future work should focus on training with data from more diverse driving environments and examining model extensions that can capture heterogeneity across drivers

